ABSTRACT

Straightforward least squares fitting of I-V curves leads to non optimal fits: residuals around and above the open-circuit voltage dominate the fit, leading to a bad fit at the maximum power point and lower voltage values. To deal with this problem authors have resorted to using weighting functions or to minimizing the area between data and fit instead of the least squares procedure. Both approaches lack a sound statistical basis.

Voltage noise has a big influence on fitting due to the steep slope of an I-V curve for higher voltage values. For this reason we have used Orthogonal Distance Regression (ODR), which is a mathematical method for fitting measurements with errors in both voltage- and current measurements. It allows for computing both the I-V curve parameters and their uncertainties. A disadvantage of this kind of methods is that not all measured points are used. It is also difficult to obtain error bounds on the parameters found. An advantage of these methods is the speed. As computers become faster, the aspect of speed becomes ever less important.

Fitting techniques

The second approach is to use fitting techniques. Let \((I_i, V_i)\) be the measured current voltage values and \(\vec{p}\) the vector of parameters to be determined. In case of the I-V curve models (1) and (2), \(\vec{p}\) consists of \(G_{sh}, R_{sh}, I_{lt}, I_{01}\) and \(n_1\) or \(I_{02}\). The optimum set of parameters \(\vec{p}\) is determined by minimizing the following sum:

\[
\vec{p}_o = \min_{\vec{p}} \sqrt{\frac{1}{n} \sum_{i=1}^{n} (w_i \cdot (I_i - I(V_i, \vec{p})))^m} \tag{4}
\]

\(n\) is the number of data points measured. The \(w_i\) are a set of weights. \(m\) determines the norm used for fitting. For \(m = 2\) we obtain standard least squares fitting. In least squares fitting the underlying assumption is that the current measurements are samples from a normal distribution with standard deviation \(\sigma(I_i)\). The \(w_i\) should then be taken as \(1/\sigma^2(I_i)\). In the case \(w_i = 1\) the fit is unweighted and we implicitly assume all the current measurements are from the same distribution.

Straightforward unweighted least squares fitting of I-V data leads to non optimal fit parameters: residuals from the open-circuit onwards dominate the fit, leading to too large residuals near the maximum power point. A typical result is shown in Fig. 1.

To deal with this problem several authors have resorted to two basically different methods.

Usage of weights

In fitting dark current measurements it is common practice to use as weights the reciprocals of the measured dark current \([4]-[6]\). For illuminated I-V curves this choice of weights can not be used since the weights become too large near \(V_{oc}\) (Martinez [7]). Zdanowicz[8] proposes sev-
Fig. 1: Unweighted least squares fit to an I-V curve. The data points have been calculated with $I_{01} = 1 \times 10^{-9}A$, $I_{02} = 2 \times 10^{-5}A$, $G_{sh} = 1.5$, $R_{se} = 7m\Omega$, $I_{0} = -3A$. A voltage noise of 1mV and a current noise of 3mA was added.

general weighting functions which give different fits. No clear arguments are presented however to chose between the different weight functions.

Usage of special norms

The higher the value of $m$ is, the more sensitive the norm is to outliers. In the extreme case of $m \to \infty$, the $m$-norm corresponds to the maximum norm. In general the least squares or 2-norm is used because it can be motivated with statistical arguments if the errors in the measurements are assumed to be normal deviates.

Chan [9] introduced minimizing the surface area between data and fit which essentially is a fit in the 1-norm, with as weights the distances between the consecutive voltage values.

Cabestany [5] used besides the standard least squares fit the maximum norm $m \to \infty$.

Datta [10] used a mixed method for a single diode model. $V_{oc}$ and $I_{sc}$ are assumed to be known. This allows $I_{0}$ and $I_{0}$ to be eliminated. The remaining parameters are determined by least squares.

Our work

Despite the large volume of literature on I-V curve fitting, we think an important aspect has been overlooked, being the influence of voltage noise as illustrated in Fig. 2.

The appreciable size of this effect is best illustrated with a numerical example based upon a 10x10cm$^2$ crystalline silicon solar cell under 1-sun illumination. We experience noise levels of around 3mA and 1mV in current and voltage respectively. The slope of the curve at $V_{oc}$ typically is 20m$\Omega$, so that the 1mV voltage noise causes a current noise level of 50mA, which is much larger than the 3mA error in the current measurement.

For this reason we have used Orthogonal Distance Regression (ODR). This is a mathematical method for fitting measurements with errors in both voltage- and current measurements. Both the choice of weights and the choice of other norms lack a sound statistical basis. ODR puts the fitted parameters on a sound statistical basis. We will describe this method and show results of the fits obtained.

ORTHOGONAL DISTANCE REGRESSION

There is a special method to find a fit in the case that there is both noise in current and voltage measurements. This method is known as Orthogonal Distance Regression (ODR) (see Boggs [11]) and can be used for instance for fitting a circle to a set of data points in a plane, indicating its ability to deal with strongly varying slopes.

We will designate noise levels in current and voltage with $\sigma(V)$ and $\sigma(I)$ respectively. With $\hat{V}_i$ and $\hat{I}_i$ we denote the fitted values.

The weighted ODR method finds a fit to the data by minimizing the following sum:

$$\min_{\hat{V}_i, \hat{I}_i, \hat{\mu}} \left[ \frac{1}{n} \sum_{i=1}^{n} \left( \frac{V_i - \hat{V}_i}{\sigma(V)} \right)^2 + \left( \frac{I_i - \hat{I}_i}{\sigma(I)} \right)^2 \right] \tag{5}$$

under the $n$ constraints that each pair $(\hat{V}_i, \hat{I}_i)$ has to be related by one of the diode-curve models from equations (1) and (2) in combination with (3). Note that for absence of voltage noise ($V_i = \hat{V}_i$), the ODR criterion (5) is equivalent to the standard least squares minimization. ODR assumes that noise in current- and voltage measurements are statistically mutually independent.

The name of the method originates from the fact that when voltage and current values are scaled with $\sigma(V)$ and $\sigma(I)$ respectively, the sum minimized by ODR consists of the sum of squares of the distances from the measured points to the noise scaled fitted I-V curve. The underlying completely statistical motivation of this method provides us with a simple criterion for the quality of the fit and allows for computing not only the diode curve parameters but also their uncertainties.

Charles [2] compared various methods of obtaining curve parameters through zero-finding on curve characteristics by looking at the sum of squares of distances from data points.
AN ODR BASED WEIGHTING SCHEME

In this section we will cast the ODR minimization procedure into a weighted least squares formulation. We observe that the total noise in a current observation consists from a part $\sigma(I)$ due to uncertainty in measuring the current and from a part $\frac{dI}{dV}\sigma(V)$ due to voltage uncertainty. We can compute a total noise level $\sigma(I(V))$ in the current measurement by adding the variances due to both noise sources:

$$\sigma(I(V))^2 = \sigma(I)^2 + \left(\frac{dI}{dV}\sigma(V)\right)^2 \tag{6}$$

The idea is that instead of a full ODR method we use a standard non-linear least squares routine to fit $I$ as a function of $V$ with $1/\sigma(I(V))$ as weights. In the following we show that the weighting scheme (6) can also be derived from ODR. The procedure is illustrated in Fig. 3.

We first transform current and voltage values as follows:

$$(I', V') = \left(\frac{I}{\sigma(I)}, \frac{V}{\sigma(V)}\right) \tag{7}$$

Let $\alpha(V')$ be the slope of the curve at voltage $V'$. $\alpha(V')$ can be related to the derivative of the $I(V)$ curve.

$$\tan(\alpha(V')) = \frac{dI'}{dV'} = \frac{\sigma(V)}{\sigma(I)} \frac{dI}{dV} \tag{8}$$

Let $\Delta I'$ be a residual in the transformed current. Because of the transformation we have $\Delta I = \sigma(I)\Delta I'$. We have for the distance $d$ to the $I' - V'$ curve:

$$d \approx \cos(\alpha)\Delta I'$$

The approximation is exact in case of a linear model. Using the trigonometrical identity $\cos^2(x) = 1/(1 + \tan^2(x))$ we obtain for the squared distance $d^2$:

$$d^2 = \cos^2(\alpha)\Delta I'^2 = \frac{\Delta^2 I'}{1 + \tan^2(\alpha)} = \frac{\Delta^2 I}{\sigma(I)^2 + \sigma(V)^2\left(\frac{dI}{dV}\right)^2} \tag{8}$$

Comparing with equation (6) we see that when the slope of the curve does not change too much, least squares fitting of $I$ as a function of $V$ with $1/\sigma(I(V))$ as a weight function is a good approximation to ODR. Also note that in practical solar cells, the slope of the curve is limited by series resistance, allowing this approach to be used. This approach can not be used in the same way to fit $V$ versus $I$, because the shunt resistance can be large rendering the $I$-$V$ curve flat for low voltage values.

The main advantage of the standard least squares formulation is that specialized Fortran routines (See Gay [12]) can be used which exploit the presence of linear parameters in the diode curve model, leading to faster and more reliable convergence. Initial estimates are needed only for the non-linear parameters. An initial estimate for the series resistance is easily obtained from the steepest slope that occurs, a good initial estimate for the ideality factor is 1.

Our computer code employs an iterative scheme such that the fitted current and not the measured current is used to calculate the effect of series resistance through equation (3).

RESULTS

Fig. 4 shows our ODR based fit to the same data as in Fig 1. Indeed the ODR fit follows the measurements more closely around the maximum power- and the short circuit point. Despite the large residuals for voltage above $V_{oc}$, the fit is still close to the data points owing to the steep slope of the curve. Fig. 4 also shows the total noise level $\sigma(I(V))$ computed with formula (6). Up to around the maximum power point at 475mV, our weighting scheme is the same as uniform weighting, since the curve is flat and errors in
The strongly varying slope makes it difficult to fit solar cell I-V curves with standard least squares techniques. Orthogonal Distance Regression (ODR) is well suited to this fitting problem and puts the fitting procedure on a sound statistical basis. In a controlled way ODR allows residuals to be larger for voltage values above $V_{oc}$, thus obtaining a better fit for lower voltage values, while still maintaining a close fit above $V_{oc}$.

The uncertainties obtained in the parameters are more realistic. The $\chi^2$ of the fit can be used to judge whether the one- or two-diode models are suitable. We have shown how ODR can be cast in the form of a weighted least squares formulation. The numerical procedure is fast and reliable enough to be used on-line.

CONCLUSIONS

This work was carried out within the framework of the ECN ENGINE programme and the Dutch NWO priority programme "Solar Cells for the 21-st century".

Table 1: Fitted curve parameters from Figs. 1 and 4 with their uncertainties

<table>
<thead>
<tr>
<th>Name</th>
<th>Uniform</th>
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<tbody>
<tr>
<td>$I_R (A)$</td>
<td>$-3.0007$</td>
</tr>
<tr>
<td>$I_{01} (A)$</td>
<td>$0.12587 \times 10^{-8}$</td>
</tr>
<tr>
<td>$I_{02} (A)$</td>
<td>$0.10572 \times 10^{-4}$</td>
</tr>
<tr>
<td>$R_{se}(\Omega)$</td>
<td>$0.77580 \times 10^{-2}$</td>
</tr>
<tr>
<td>$G_{sh}(S)$</td>
<td>$0.14337$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Name</th>
<th>ODR</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_R (A)$</td>
<td>$-3.0001$</td>
</tr>
<tr>
<td>$I_{01} (A)$</td>
<td>$0.10410 \times 10^{-8}$</td>
</tr>
<tr>
<td>$I_{02} (A)$</td>
<td>$0.18997 \times 10^{-4}$</td>
</tr>
<tr>
<td>$R_{se}(\Omega)$</td>
<td>$0.71995 \times 10^{-2}$</td>
</tr>
<tr>
<td>$G_{sh}(S)$</td>
<td>$0.10354$</td>
</tr>
</tbody>
</table>

The voltage contribute little to errors in the current. From the maximum power point onwards, the influence of voltage noise increases sharply. This is explained by the fact that for higher voltages the total noise according to (6) can be approximated with:

$$\sigma_i(I(V)) = \frac{df}{dV} \sigma(V)$$

Table 1 compares the results from both fits. The results can be compared with the known parameters of the I-V curve from Fig. 1. The ODR fit returns much better estimates of the I-V curve parameters and gives smaller and more accurate uncertainty levels. In the unweighted fit $G_{sh}$ and $I_{02}$ are most off.

A further conclusion, which is supported by calculations not shown here is that the fit is less sensitive to the range of voltage values used.

ACKNOWLEDGMENTS

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REFERENCES


